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Viscous Dissipation Effects on MHD Flow over a Vertical Plate with Ramped Wall Temperature and Chemical Reaction in the Presence of Radiation

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Abstract: **In this paper, the numerical solution of viscous dissipation effects on unsteady magneto-hydrodynamic (MHD) free convection flow of an incompressible viscous, electrically conducting fluid near an infinite vertical plate with ramped wall temperature and chemical reaction in the presence of radiation has been carried out. The motion of the plate is a rectilinear translation with an arbitrary time dependent velocity and the radiative flux is described by using differential approximation. The partial differential equations governing the flow are transformed into a non-dimensional form and are solved by Ritz finite element method. The effects of the flow parameters on the velocity, temperature and concentration fields are presented through the graphs and numerical data for the skin-friction presented in table. The results obtained are discussed for two cases, namely when the magnetic field is fixed to the fluid and moving plate.**

Keywords: **MHD, free convection, ramped wall temperature, chemical reaction, viscous dissipation.**

I. INTRODUCTION

Radiative convection flows are encountered in many areas of industrial and environmental processes. e.g., heating and cooling chambers, fossil fuel combustion energy processes, evaporation for large open water reservoirs, astrophysical flows, solar power technology and space vehicle re-entry. Also, many areas of technology and applied physics including oxide melt materials processing, astrophysical fluid dynamics, plasma flows switch performance, MHD energy pumps operating at very high temperatures and hypersonic aerodynamics. Raptis [1] studied the flow of a micro polar fluid past a continuously moving plate in the presence of radiation. Thermal radiation effects on unsteady MHD free convection flow past a vertical plate with temperature dependent viscosity presented by Mohamoud [2]. Shanaker et. al [3] presented the radiation and mass transfer effects on unsteady MHD free convective fluid flow embedded in a porous medium with heat generation/absorption. Reddy and Rao [4] studied heat and mass transfer of an unsteady MHD natural convection flow of a rotating fluid past a vertical porous plate in the presence of radiative heat transfer by finite element method. Seth et. al [5] studied MHD natural convection flow with radiative heat transfer past an impulsively moving plate with ramped wall temperature. Ghara et. al [6] presented the effect of radiation on MHD free convection flow past an impulsively moving vertical plate with ramped wall temperature. Ahmed and Dutta [7] studied transient mass transfer flow past an impulsively started infinite vertical plate with ramped plate velocity and ramped wall temperature. Narahari and Debnath [8] studied unsteady magneto-hydrodynamic free convection flow past an accelerated vertical plate with constant heat flux and heat generation/absorption. Unsteady magneto-hydrodynamic free convection flow of a second grade fluid in a porous medium with ramped wall temperature studied by Samiulhaq et. al [9]. Seddek [10] presented the effects of chemical reaction, variable viscosity, thermo-phoresis and heat generation/absorption on a boundary-layer hydro-magnetic flow with heat and mass transfer over a heat surface by finite element method. Seddek et. al [11] presented the effects of chemical reaction and variable viscosity on hydro-magnetic mixed convection heat and mass transfer for Hymens flow through porous media with radiation. The effects of chemical reaction and radiation absorption on the unsteady MHD free convection flow past a semi-infinite vertical permeable moving plate with heat source and suction presented by Ibrahim et. al [12]. Patil and Kulkarni [13] presented the effects of chemical reaction on free convective flow of a polar fluid through a porous medium in the presence of internal heat generation.

The viscous dissipation effect is expected to be relevant for fluids with high values of dynamic viscosity as for high velocity flows. The viscous dissipation heat is important in the natural convective flows, when the field is of extreme size or at extremely low temperature or in high gravitational field. Gebhart [14] has shown the importance of viscous dissipative heat in free convection flow in the case of isothermal and constant heat flux at the plate. Gebhart and Mollendorf [15] presented the effects of viscous dissipation for external natural convection flow over a surface. Soundalgekar [16] analyzed the viscous dissipation heat on the two-dimensional unsteady free convective flow past an infinite vertical porous plate when the temperature oscillates in time and there is constant suction at the plate. Cookey et al. [17] have studied the influence of viscous dissipation and radiation on unsteady MHD free convection flow past an infinite heated vertical plate in a porous medium with time dependent suction. Recently, Reddy [18] presented the mass transfer effects on unsteady MHD free convective flow of an incompressible viscous dissipative fluid past an infinite vertical porous plate. Reddy [19] presented the viscous dissipation effects on unsteady hydro-magnetic gas flow along an inclined plane with indirect natural convection in the presence of thermal radiation.

Hence, based on the above investigations and applications, the objective of the present paper is to analyze the viscous dissipation effects on unsteady MHD free convection flow of an incompressible, electrically conducting fluid near an infinite vertical plate with ramped wall temperature and chemical reaction in the presence of radiation. The Ritz finite element method has been adopted to solve the system of partial differential equations which is more economical from computational point of view. The fluid is electrically conducting and regarding the applied magnetic field two cases are considered, namely, when the magnetic lines of force are fixed to the fluid and plate. The differences between fluid velocities in the two cases are studied and some properties are highlighted.

II. MATHEMATICAL MODEL

A two dimensional unsteady magneto-hydrodynamic (MHD) free convection flow of viscous incompressible, electrically conducting fluid near an infinite vertical plate with ramped wall temperature in the presence of radiation is considered. The motion of the plate is a rectilinear translation with an arbitrary time dependent velocity. We introduce a coordinate system with $x'-$ axis along the plate in the vertical upward direction, and $y'-$ axis normal plate. A uniform transverse magnetic field of strength B_0 is applied. Initially, at time $t'=0$ the plate and the fluid are at rest with the same temperature T_∞ the species concentration in the fluid C_∞. After time $t'=0^+$ the plate moves with the velocity $U_0 f(t')$ in its own plane along the $x'-$ axis. Here, U_0 is a constant velocity and $f(\cdot)$ is a dimensionless piecewise continuous function whose values $f(0)=0$. Heat is supplied to the plate as a time-ramped function in the presence of chemical reaction. The species concentration at the plate is C_w. The magnetic Reynolds number is small so that the induced magnetic field is negligible in comparison to the applied magnetic field. No external electric field is applied and the effect of polarization of ionized fluid negligible, therefore, electric field is assumed to be zero. There exists a first order chemical reaction between the fluid and species concentration. Since the plate is infinite extended in x and z directions, therefore all the physical quantities are functions of the spatial coordinate y' and t' only. Then, under the Boussinesq's approximation, the flow governed by the following system of equations:

$$\frac{\partial u'}{\partial t'} = g\beta_T\left(T'-T_\infty\right) + g\beta_C\left(C'-C_\infty\right) + \nu\frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho}u' \tag{1}$$

$$\rho C_p\frac{\partial T'}{\partial t'} = k\frac{\partial^2 T'}{\partial y'^2} - \frac{\partial q_r'}{\partial y'} + \nu\left(\frac{\partial u'}{\partial y'}\right)^2 \tag{2}$$

$$\frac{\partial C'}{\partial t'} = D_m \frac{\partial^2 C'}{\partial y'^2} - k_r' \left(C' - C_\infty \right) \tag{3}$$

where u', T' and C' are velocity, temperature and species concentration of the fluid, respectively, ν is kinematic viscosity of the fluid, g is the acceleration due to gravity, ρ is the fluid density, C_p is the specific heat at constant pressure, k is the thermal conductivity of the fluid, D_m is the chemical molecular diffusivity, β_T is the volumetric coefficient of thermal expansion, β_C is the volumetric coefficient of concentration expansion, k_r' is the chemical reaction parameter, B_0 is the uniform magnetic field, t' is the time.

Equation (1) is valid, when the magnetic lines of force are fixed relative to the fluid. If the magnetic field is fixed to the plate, the momentum equation (1) is replaced by [8, 20]

$$\frac{\partial u'}{\partial t'} = g\beta_T \left(T' - T_\infty \right) + g\beta_C \left(C' - C_\infty \right) + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} \left(u' - U_0 f(t) \right) \tag{4}$$

Equations (1) and (4) combined as:

$$\frac{\partial u'}{\partial t'} = g\beta_T \left(T' - T_\infty \right) + g\beta_C \left(C' - C_\infty \right) + \nu \frac{\partial^2 u'}{\partial y'^2} - \frac{\sigma B_0^2}{\rho} \left(u' - \varepsilon U_0 f(t) \right) \tag{5}$$

where

$\varepsilon = 0$ if B_0 is fixed relative to the fluid

$= 1,$ if B_0 is fixed relative to the plate.

The corresponding initial and boundary conditions are:

$t' \le 0 : u' = 0, T' = T_\infty, C' = C_\infty,$ for all $y' \ge 0$

$$t' > 0 : u' = U_0 f(t), T' = \begin{cases} T_\infty + (T_w - T_\infty)\dfrac{t'}{t_0}, 0 < t' \le t_0 \\ T_w, t' > t_0 \end{cases}, \; C' = C_w$$

$$u' < \infty, T' \to T_\infty, C' \to C_\infty \text{ as } y' \to \infty \tag{6}$$

By using the Rosseland approximation [1], the radiative flux vector q_r' can be written as:

$$q_r' = -\frac{4\sigma^*}{3k^*} \frac{\partial T'^4}{\partial y'} \tag{7}$$

It is assumed that the temperature differences within the flow are sufficiently small so that T'^4 can be expanded in a Taylor series about the free stream temperature T_∞, so that after neglecting the higher order terms

$$T'^4 \approx 4T_\infty^3 T' - 3T_\infty^4 \tag{8}$$

The energy equation after substitution of equations (7) and (8) can be written as:

$$\frac{\partial T'}{\partial t'} = \frac{k}{\rho C_p} \frac{\partial^2 T'}{\partial y'^2} + \frac{16\sigma^* T_\infty^3}{3\rho C_p k^\bullet} \frac{\partial^2 T'}{\partial y'^2} + \frac{\nu}{\rho C_p} \left(\frac{\partial u'}{\partial y'} \right)^2 \tag{9}$$

It is convenient to introduce the following non-dimensional quantities into the basic equation, initial and boundary conditions in order to make them dimensionless.

$$u=\frac{u'}{U_0}, y=\frac{y'}{\sqrt{\nu t_0}}, t=\frac{t'}{t_0}, \theta=\frac{T'-T_\infty}{T_w-T_\infty}, C=\frac{C'-C_\infty}{C_w-C_\infty}, S_c=\frac{\nu}{D_m}, P_r=\frac{\mu C_p}{k}, M=\sqrt{\nu t_0}B_0\sqrt{\frac{\sigma}{\mu}},$$

$$k_r=k_r^{'}t_0, R=\frac{16\sigma^* T_\infty^3}{3k^\bullet k}, E_c=\frac{t_0^2}{C_p(T_w^{'}-T_\infty^{'})}, G_r=\frac{g\beta_T t_0(T_w-T_\infty)}{U_0}, G_m=\frac{g\beta_C t_0\left(C_w-C_\infty\right)}{U_0}.$$

After substituting the above non-dimensional quantities into equations $(3),(5),(6)$ and (9), we obtain the governing equations in non-dimensional form are:

$$\frac{\partial u}{\partial t}=\frac{\partial^2 u}{\partial y^2}+G_rT+G_mC-M^2\left(u-\varepsilon f(t)\right) \tag{10}$$

$$\frac{\partial \theta}{\partial t}=\left(\frac{1+R}{P_r}\right)\frac{\partial^2 \theta}{\partial y^2}+E_c\left(\frac{\partial u}{\partial y}\right)^2 \tag{11}$$

$$\frac{\partial C}{\partial t}=\frac{1}{S_c}\frac{\partial^2 C}{\partial y^2}-k_rC \tag{12}$$

where G_r is the thermal Grashof number, G_m is the mass Grashof number, M is the magnetic parameter, R is the radiation parameter, P_r is the Prandtl number, E_c is the Eckert number, S_c is the Schmidt number and k_r is the chemical reaction parameter.

The corresponding initial and boundary conditions are:

$t\le 0; u=0, T=0, C=0,$ for all $y\ge 0$

$$t>0; u=f(t), T=\begin{cases} t, 0\le t\le 1\\ 1, t>1\end{cases}=tH(t)-(t-1)H(t-1), C=1 \text{ for } y=0$$

$$u<\infty, T\to 0, C\to 0 \text{ as } y\to\infty \tag{13}$$

where

$$H(t)=\begin{cases}0, t\le 0\\ 1, t>0\end{cases}$$ is the Heaviside unit step function.

III. SOLUTION OF THE PROBLEM

Equations $(10)-(12)$ are non-linear systems of partial differential equations are solved under the initial and boundary conditions given in equation (13). However, whose exact or approximate solutions are not possible. Hence, the Ritz finite element method applied to solve these equations. The Ritz finite element method has been employed extensively by the authors in many challenging heat and mass transfer, biomechanics and metallurgical transport phenomena problems over the past few years. The method entails the following steps.

1) Division of the whole domain into smaller elements of finite dimensions called “finite elements”.
2) Generation of the element equations using variational formulations.
3) Assembly of element equations as obtained in step 2.
4) Imposition of boundary conditions to the equations obtained in step 3.
5) Solution of the assembled algebraic equations.

The assembled equations can be solved by any of the numerical technique viz. Gauss-Seidal iteration method. Here, $y\to\infty$ is taken as $y_{max}=10$. An important consideration is that of shape functions which are employed to approximate actual functions. For one and two dimensional problems, the shape functions can be linear/quadratic and higher order. However, the suitability of the shape functions varies from problem to problem. Due to simple and efficient use in computations, linear shape functions are used in the present problem. To judge the accuracy of convergence and stability of the Ritz finite element method, the computations are carried out by making small changes time t and $y-$directions. For these slightly changed values, no significant change was observed in the values of velocity, temperature and concentration. Hence, we conclude that the Ritz finite element method is convergent and stable.

The skin-friction at the plate surface $(y=0)$ is given by $\tau=\left(\frac{\partial u}{\partial y}\right)_{y=0}$

IV. NUMERICAL RESULTS AND DISCUSSION

The problem of viscous dissipation effects on unsteady MHD free convection flow of an incompressible, electrically conducting fluid near an infinite vertical plate with ramped wall temperature and chemical reaction in the presence of radiation is addressed in this study. Numerical calculations have been carried out for the velocity (u), temperature (θ), species concentration (C) and skin-friction (τ) for various values of material parameters encountered in the problem under the investigation. The numerical calculations of these results are presented through graphs and table. The results obtained are discussed for the magnetic lines of force are fixed relative to the fluid and moving plate.

The effects of the Prandtl number P_r, radiation parameter R and Eckert number E_c on the temperature field are presented in Figs 1 and 2, respectively when the magnetic field is fixed to the fluid and moving plate. In both the cases, it is observed that an increase in the Prandtl number leads to decrease in the temperature field whereas an increase in the radiation parameter and Eckert number leads to increase in the temperature field. Figure 3 depicts the effects of Schmidt number S_c and chemical reaction parameter k_r on the concentration field. It is observed that an increase in the Schmidt number and chemical reaction parameter leads to decrease in the concentration field due to the decrease in the molecular diffusivity results a decrease in the concentration boundary layer.

The velocity profiles versus the spatial variable y for constant plate velocity $(f(t)=H(t))$ are presented in figures 4-11, respectively. The figures, corresponding to the velocity field are plotted when the magnetic field is being fixed to the fluid $(\varepsilon=0.0)$ and to the moving plate $(\varepsilon=1.0)$. Figure 4 depicts the effect of the Prandtl number P_r on the velocity field. It can be seen clearly that an increase in the Prandtl number leads to decrease in the fluid velocity. This is due to the fact that when the Prandtl number increase, thermal conductivity of the fluid decreases, that causes the reduction in the fluid velocity. The effects of the radiation parameter R and viscous dissipation parameter i.e., Eckert number E_c on the velocity field are presented in figures 5 and 6, respectively. It is observed that an increase in the radiation parameter and viscous dissipation parameter increases in the fluid velocity. Figs. 7 and 8 shows the effects of Schmidt number S_c and chemical reaction parameter k_r on the velocity field, respectively. It is seen that an increasing value of the Schmidt number and chemical reaction parameter decreases the fluid velocity. Figure 9 depicts the effect of magnetic parameter M on the velocity field. It is observed that fluid velocity decreases as the magnetic parameter increases. Due to the fact that, under the influence of magnetic field on an electrically conducting fluid, a resistive force arises (so called the Lorentz force). This force has tendency to slow down the fluid motion in the boundary layer. Figure 10 displays the velocity field for various values of the thermal Grashof number G_r. It is noticed that the velocity increases with increasing values of thermal Grashof number. This is due to the presence of thermal buoyancy that enhances the fluid velocity. Figure 11 depicts the effect of mass Grashof number G_m on the velocity field. It is observed that the fluid velocity increases with increasing values of mass Grashof number. This is due to the presence of mass buoyancy that enhances the fluid velocity. We notice that, the fluid velocity has a maximum value in the vicinity of the plate and tends to the finite value for larger values of the spatial coordinate y. Further, it is noted that if the magnetic field is fixed to the fluid $(\varepsilon=0.0)$, the values of the fluid velocity are lower than in case of the magnetic field is fixed to the moving plate $(\varepsilon=1.0)$.

The numerical data for ski-friction coefficient (τ) for variations in Prandtl number, radiation parameter, Eckert number, Schmidt number, chemical reaction parameter, magnetic parameter, thermal Grashof number and mass Grashof number is presented in table 1, when the magnetic field is fixed to the fluid $(\varepsilon = 0.0)$ and the plate $(\varepsilon = 1.0)$, respectively. In both the cases, it is observed that an increase in the Prandtl number, Schmidt number, chemical reaction parameter and magnetic parameter decreases the value of skin-friction coefficient whereas an increase in the radiation parameter, Eckert number, thermal Grashof number and mass Grashof number increases the value of skin-friction coefficient. Also, it is noted that if the magnetic field is fixed to the fluid $(\varepsilon = 0.0)$, the numerical values of the skin-friction coefficient are lower than in case of the magnetic field is fixed to the moving plate $(\varepsilon = 1.0)$.

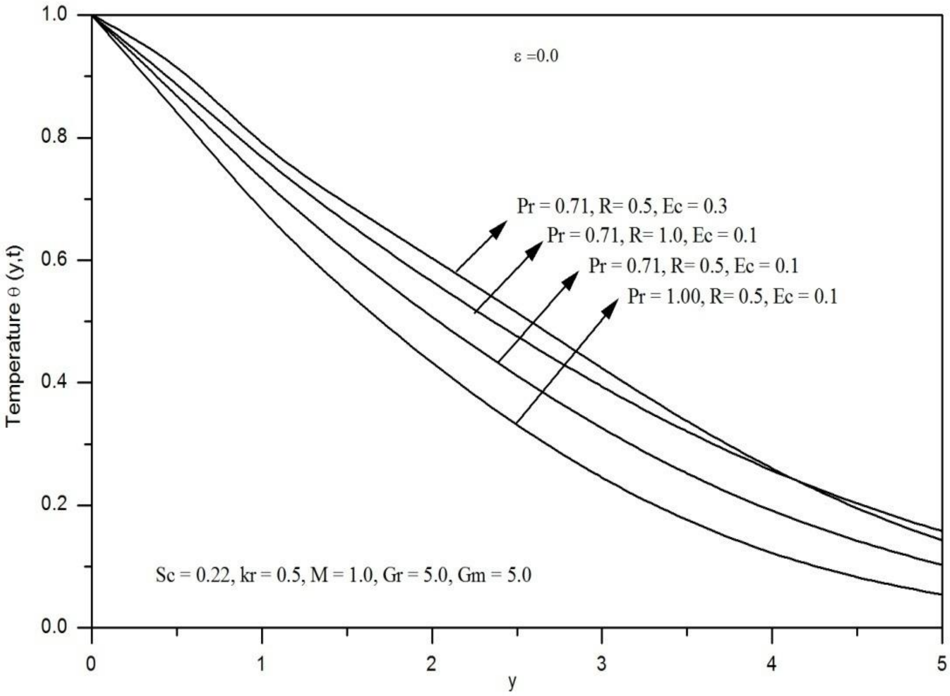

Figure 1: Temperature profiles for $\varepsilon = 0.0$

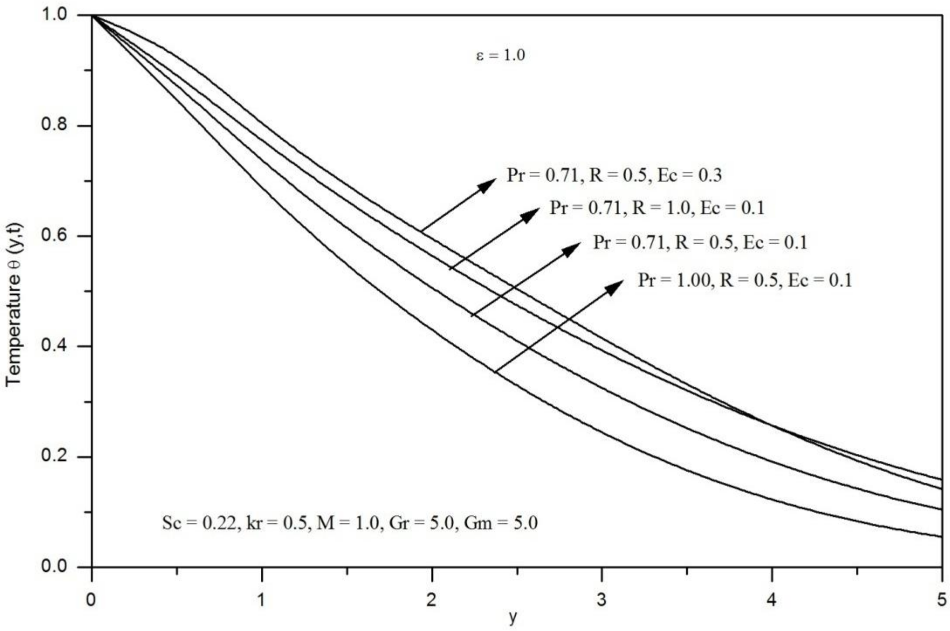

Figure 2: Temperature profiles for $\varepsilon = 1.0$

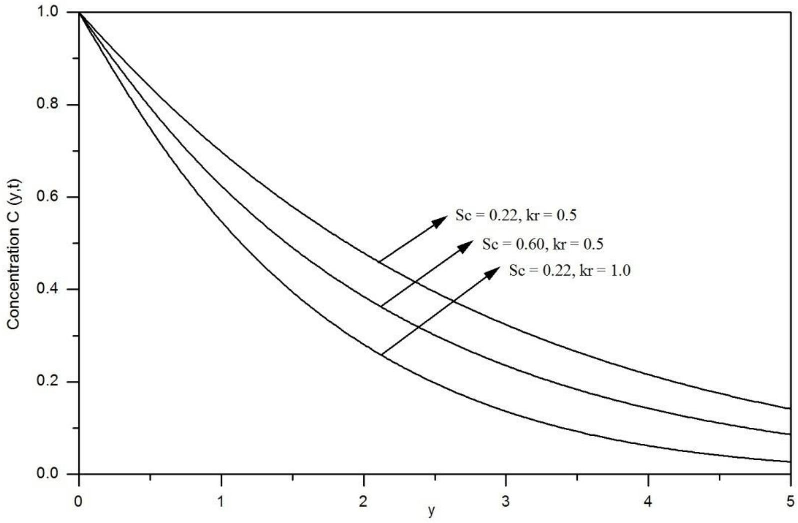

Figure 3: Concentration profiles

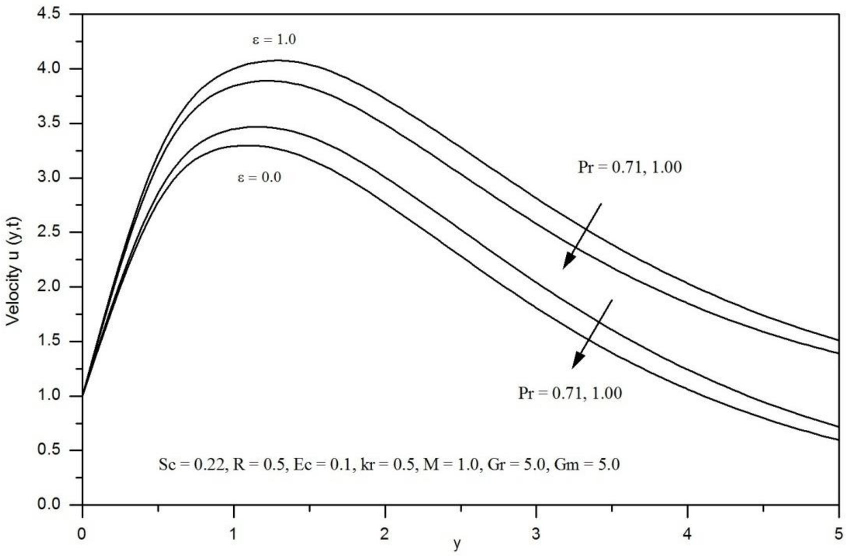

Figure 4: Effect of Prandtl number P_r on the velocity field

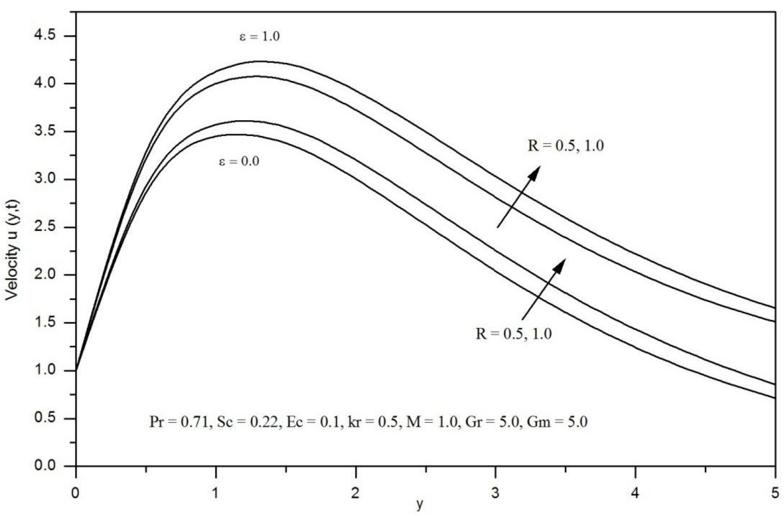

Figure 5: Effect of radiation parameter R on the velocity field

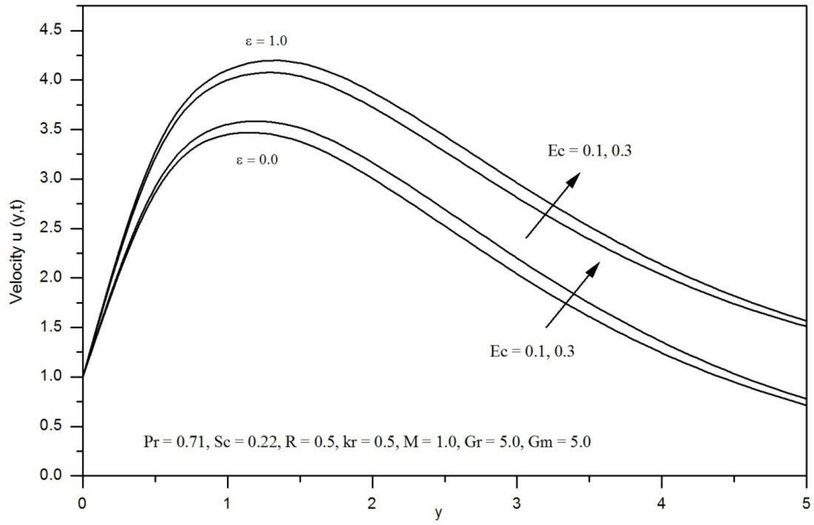

Figure 6: Effect of Eckert number E_c on the velocity field

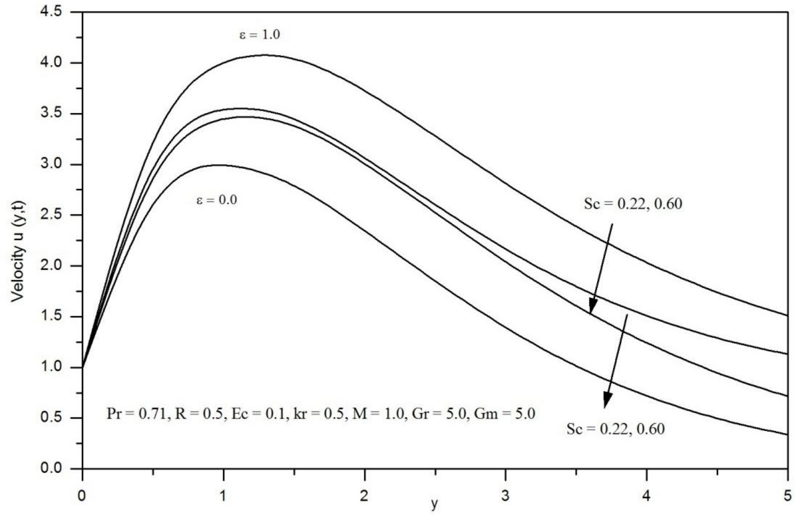

Figure 7: Effect of Schmidt number S_c on the velocity field

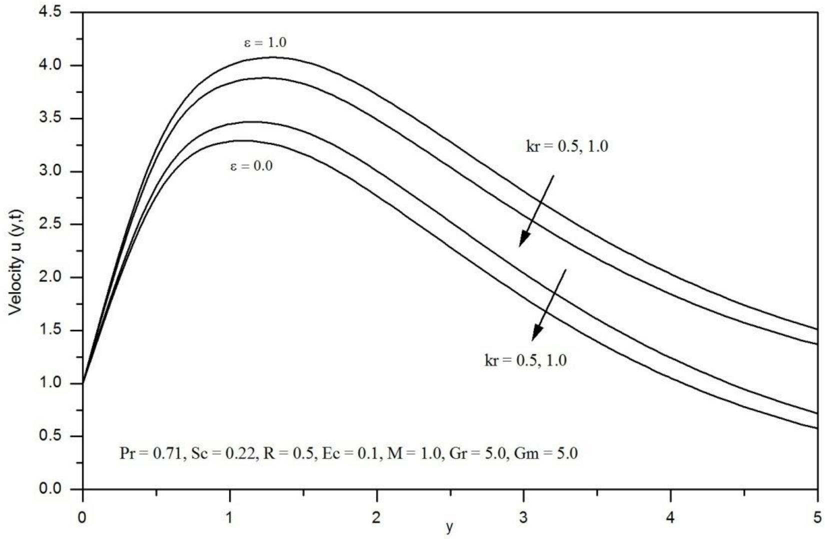

Figure 8: Effect of chemical reaction parameter k_r on the velocity field

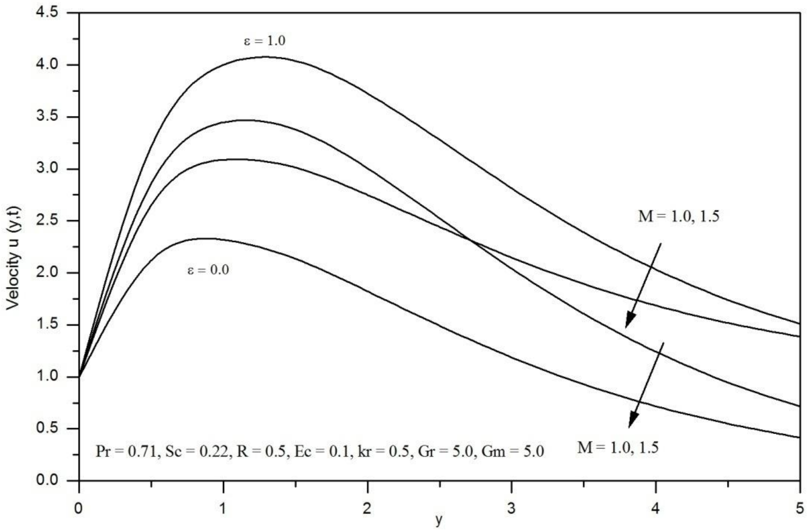

Figure 9: Effect of magnetic parameter M on the velocity field

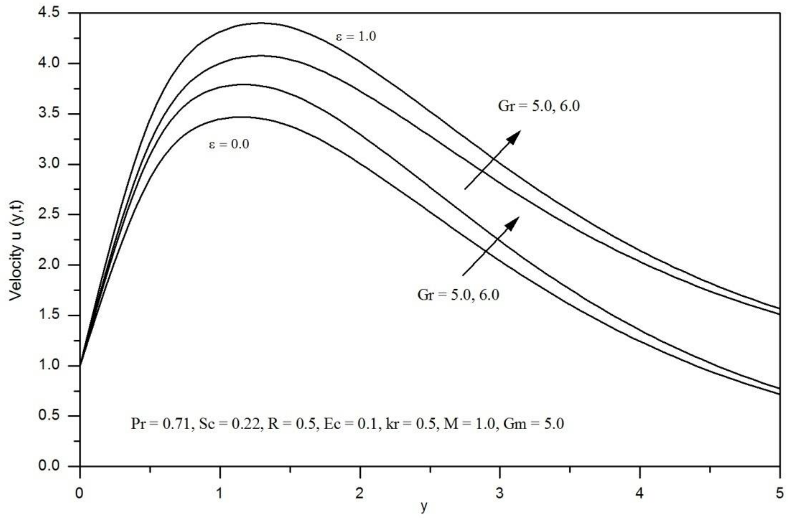

Figure 10: Effect of thermal Grashof number G_r on the velocity field

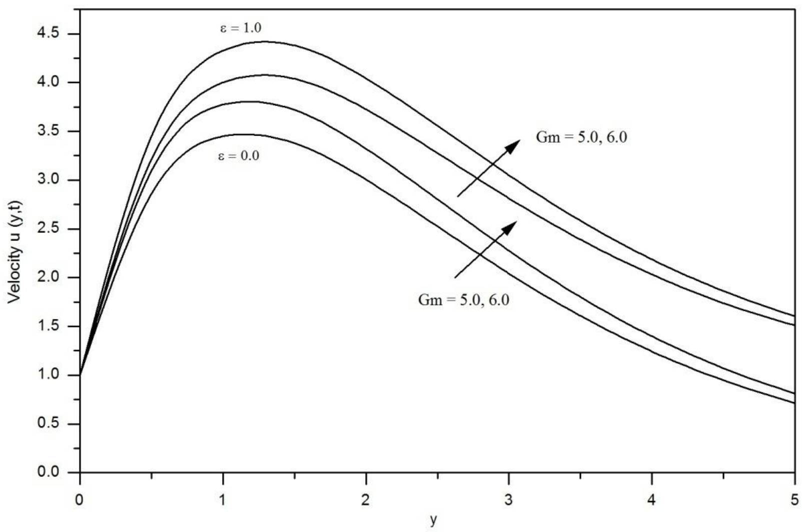

Figure 11: Effect of mass Grashof number G_m on the velocity field

Table 1: Numerical data for the skin-friction coefficient (τ) when, magnetic field is fixed to the fluid $(\varepsilon = 0.0)$ and moving plate $(\varepsilon = 1.0)$.

P_r	R	E_c	S_c	k_r	M	G_r	G_m	$\tau(\varepsilon = 0.0)$	$\tau(\varepsilon = 1.0)$
0.71	0.5	0.1	0.22	0.5	1.0	5.0	5.0	3.723770	4.432466
1.00	0.5	0.1	0.22	0.5	1.0	5.0	5.0	3.554388	4.261686
0.71	1.0	0.1	0.22	0.5	1.0	5.0	5.0	3.856336	4.565718
0.71	0.5	0.3	0.22	0.5	1.0	5.0	5.0	3.838852	4.559854
0.71	0.5	0.1	0.60	0.5	1.0	5.0	5.0	3.212534	3.913830
0.71	0.5	0.1	0.22	1.0	1.0	5.0	5.0	3.538274	4.244942
0.71	0.5	0.1	0.22	0.5	1.5	5.0	5.0	2.245600	3.303024
0.71	0.5	0.1	0.22	0.5	1.0	6.0	5.0	4.184292	4.895266
0.71	0.5	0.1	0.22	0.5	1.0	5.0	6.0	4.194400	4.904802

V. CONCLUSIONS

The governing equations of the flow are analyzed for the effect of viscous dissipation on unsteady magneto-hydrodynamic (MHD) free convection flow of an incompressible, electrically conducting fluid near an infinite vertical porous plate with ramped wall temperature and chemical reaction in the presence of radiation. The motion of the plate is a rectilinear translation with an arbitrary time dependent velocity, the plate temperature changes as a time-ramped function. The Ritz finite element method has been adopted to solve the governing equations of the flow. The results obtained are discussed for the magnetic field that is fixed relative to the fluid $(\varepsilon = 0.0)$ and moving plate $(\varepsilon = 1.0)$. The fluid velocity differs significantly, when the magnetic field is fixed relative to the moving plate from the fluid velocity corresponding to the case of magnetic field is fixed relative to the fluid. The fluid velocity increases with increasing values of the viscous dissipation parameter, radiation parameter, thermal Grashof number and mass Grashof number and decreases with an increase in Prandtl number, Schmidt number and chemical reaction parameter. An increase in the magnetic field decreases the fluid velocity. i.e., stronger magnetic field leads to slower flows. The magnetic field is fixed to the fluid $(\varepsilon = 0.0)$, the values of the fluid velocity are lower than in the case of the magnetic field is fixed to the moving plate $(\varepsilon = 1.0)$.

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